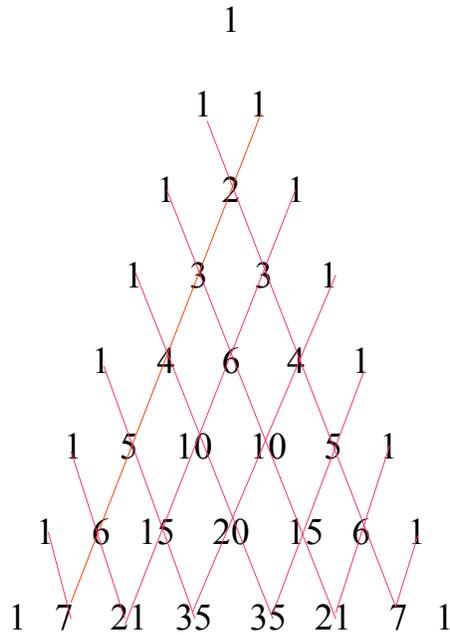


# Pascal's Triangle

Pascal's triangle shows the coefficients of each term in the expansion of a binomial of the form  $(x + y)^n$ .

## Pascal's Triangle



Each row begins and ends with a 1. The terms in between the 1's are the sum of the two terms directly above their location diagonally to the left and right. The pattern continues on indefinitely although this example only has the first 8 rows. The diagonal lines are not actually part of the triangle they are just put there so it will be easy to see the pattern. They are also helpful when drawing Pascal's triangle.

Look at the triangle. In the 3<sup>rd</sup> row, 1 2 1, the 2 is the sum of the 1 above it diagonally to its left and to its right. In the 4<sup>th</sup> row, 1 3 3 1, the 3 on the left is the sum of the 1 and 2 diagonally above it to the left and the right. The same is true for the second 3. In the 5<sup>th</sup> row, 1 4 6 4 1, the 4 on the left is the sum of the 1 and 3 diagonally above it to its left and right. The 6 is the sum of the 3 and 3 diagonally above it to its left and right. Lastly the 4 is the sum of the 3 and 1 diagonally above it to its left and right. The pattern continues on indefinitely.

An easier way than using Pascal's triangle to find the coefficients of the terms of the expansion is to evaluate the expression  $\binom{n}{k-1}$  where  $n$  is the exponent on the expansion and  $k$  is the number of the term. So if you want the 3<sup>rd</sup> term in the expansion of  $(x + y)^7$  then  $n=7$  and  $k=3$ . You would evaluate  $\binom{7}{3-1} = \binom{7}{2} = 21$ .



Remember this pattern: the top number is the exponent on the expansion, the bottom number is 1 less than the number of the terms position. If you remember this pattern you can just write the term and evaluate it without having to draw Pascal's triangle.

Here are some more examples without the use of Pascal's triangle:

1. What is the 9<sup>th</sup> term in the expansion of  $(x + y)^{12}$  ? To draw Pascal's triangle to solve this would require that 13 rows are drawn. If this second method is used the only work that needs to be done is to figure out the value of  $\binom{12}{8}$ . Remember the top number is the exponent on the expansion and the bottom number is one less than the terms position in the expansion. If I need to find the coefficient of the 9<sup>th</sup> term in an expansion that is raised to the 12<sup>th</sup> power I need to use 8 on the bottom and 12 on top.  $\binom{12}{8}$  evaluates to 495.
2. What is the 10<sup>th</sup> term in the expansion of  $(x + y)^{12}$  ? The only work that needs to be done is to figure out the value of  $\binom{12}{9}$ . Remember the top number is the exponent on the expansion and the bottom number is one less than the term number. If I need to find the coefficient of the 10<sup>th</sup> term in an expansion that is raised to the 12<sup>th</sup> power I need to use 9 on the bottom and 12 on the top.  $\binom{12}{9}$  evaluates to 220.
3. What is the 23<sup>th</sup> term in the expansion of  $(x + y)^{25}$  ? The only work that needs to be done is to figure out the value of  $\binom{25}{22}$ . Remember the top number is the exponent on the expansion and the bottom number is one less than the term number. If I need to find the coefficient of the 23<sup>rd</sup> term in a expansion that is raised to the 25<sup>th</sup> power I need to use 22 on the bottom and 25 on the top.  $\binom{25}{22}$  evaluates to 300. There is no way I would want to draw Pascal's triangle to figure this one out.

Not only can this method be used to figure out the coefficient of a single term it can be used to figure out the coefficients of all the terms in a binomial expansion. It is much faster than drawing Pascal's triangle and certainly much faster and less prone to errors than actually expanding the binomial.